Calculus BC - Analytic Limits ws

① $\lim_{x \to 0} \frac{5x^3 + 8x^2}{3x^4 - 16x^2}$

$\lim_{x \to 0} \frac{\cancel{x^2}(5x + 8)}{\cancel{x^2}(3x^2 - 16)}$

$= -\frac{8}{16} = \boxed{-\frac{1}{2}}$

② $\lim_{x \to 5} \frac{\frac{2}{x+3} - \frac{1}{4}}{x-5}$

$\lim_{x \to 5} \frac{8 - (x+3)}{4(x+3)} \cdot \frac{1}{x-5}$

$\lim_{x \to 5} \frac{-x+5}{4(x+3)(x-5)}$

$\lim_{x \to 5} \frac{-(\cancel{x-5})}{4(x+3)(\cancel{x-5})} = \frac{-1}{4(5+3)}$

$= \boxed{-\frac{1}{32}}$

③ $\lim_{x \to 0} \frac{(2+x)^3 - 8}{x}$

$\lim_{x \to 0} \frac{x^3 + 6x^2 + 12x + 8 - 8}{x}$

$\lim_{x \to 0} x^2 + 6x + 12$

$\boxed{12}$

④ $\lim_{y \to -3} \frac{y^2 + 4y + 3}{y^2 - 3}$

$= \frac{9 - 12 + 3}{9 - 3}$

$= \frac{0}{6} = \boxed{0}$

⑤ $\lim_{t \to 2} \frac{t^3 + 2t^2 - 13t + 10}{t^3 + 4t^2 - 4t - 16}$

$\lim_{t \to 2} \frac{(\cancel{t-2})(t^2 + 4t - 5)}{(\cancel{t-2})(t^2 + 6t + 8)}$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -13 & 10 \\ & & 2 & 8 & -10 \\ \hline & 1 & 4 & -5 & 0 \end{array}$$

$= \frac{4 + 8 - 5}{4 + 12 + 8} = \boxed{\frac{7}{24}}$

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -4 & -16 \\ & & 2 & 12 & 16 \\ \hline & 1 & 6 & 8 & 0 \end{array}$$

⑥ $\lim_{\Delta x \to 0} \frac{2(3 + \Delta x)^3 - 54}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2((\Delta x)^3 + 9(\Delta x)^2 + 27\Delta x + 27) - 54}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{2(\Delta x)^3 + 18(\Delta x)^2 + 54\Delta x}{\Delta x}$

$\lim_{\Delta x \to 0} 2(\Delta x)^2 + 18\Delta x + 54$

$= \boxed{54}$

⑦ $\lim_{h\to 0} \frac{4(x+h)^2 - 3(x+h)+5 - (4x^2-3x+5)}{h}$

$\lim_{h\to 0} \frac{\cancel{4x^2} + 8xh + 4h^2 - \cancel{3x} - 3h + \cancel{5} - \cancel{4x^2} + \cancel{3x} + \cancel{5}}{h}$

$\lim_{h\to 0} \frac{8xh + 4h^2 - 3h}{h}$

$\lim_{h\to 0} 8x + 4h - 3$

$= \boxed{8x - 3}$

⑧ $\lim_{x\to 3} \frac{\sqrt{x+6} - 3}{x-3}\left(\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right)$

$\lim_{x\to 3} \frac{x+6-9}{(x-3)(\sqrt{x+6}+3)}$

$\lim_{x\to 3} \frac{1}{\sqrt{x+6}+3}$

$= \boxed{\frac{1}{6}}$

⑨ $\lim_{x\to 1} \frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$

$\lim_{x\to 1} \frac{(2x-3)(x-1)}{(2x^2+x-3)(\sqrt{x}+1)}$

$\lim_{x\to 1} \frac{(2x-3)\cancel{(x-1)}}{(2x+3)\cancel{(x-1)}(\sqrt{x}+1)}$

$= \frac{-1}{5\cdot 2} = \boxed{-\frac{1}{10}}$

⑩ $g(x) = \begin{cases} 5-2x, & x>1 \\ 4, & x=1 \\ 4-x, & x<1 \end{cases}$

a) $\lim_{x\to 5} g(x) = 5-2(5) = -5$

b) $\lim_{x\to 1^-} g(x) = 4-1 = 3$

c) $\lim_{x\to 1^+} g(x) = 5-2(1) = 3$

d) $\lim_{x\to 1} g(x) = 3$ (left = right)

⑪ $\lim_{\Delta x\to 0} \frac{\cos(x+\Delta x) - \cos x}{\Delta x}$ (Note: x is a constant)

$\lim_{\Delta x\to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cos x(\cos \Delta x - 1) - \sin x \sin \Delta x}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cos x(\cos \Delta x - 1)}{\Delta x} - \lim_{\Delta x\to 0} \frac{\sin x \cdot \sin \Delta x}{\Delta x}$

$\cos x \cdot \lim_{\Delta x\to 0} \frac{\cos \Delta x - 1}{\Delta x} - \sin x \cdot \lim_{\Delta x\to 0} \frac{\sin \Delta x}{\Delta x}$

$= \cos x\,(0) - \sin x \cdot 1$

$= \boxed{-\sin x}$

(12) $\lim_{x \to 0} \dfrac{\cot 4x}{\cot 3x}$

$\lim_{x \to 0} \dfrac{\frac{\cos 4x}{\sin 4x}}{\frac{\cos 3x}{\sin 3x}}$

$\lim_{x \to 0} \dfrac{\cos 4x}{\sin 4x} \cdot \dfrac{\sin 3x}{\cos 3x}$

$\lim_{x \to 0} \left(\dfrac{4x}{\sin 4x}\right)\left(\dfrac{\sin 3x}{3x}\right)\left(\dfrac{\cos 4x}{\cos 3x}\right)\left(\dfrac{3x}{4x}\right)$

$= 1 \cdot 1 \cdot \dfrac{\cos 0}{\cos 0} \cdot \dfrac{3}{4}$

$= \boxed{\dfrac{3}{4}}$

(13) $\lim_{x \to 0} \dfrac{\sin x}{5x^2 - x}$

$\lim_{x \to 0} \dfrac{\sin x}{x} \cdot \dfrac{1}{5x-1}$

$= 1 \cdot \dfrac{1}{-1} = \boxed{-1}$

(14) $\lim_{x \to 0} \dfrac{4x + \sin 2x}{x}$

$\lim_{x \to 0} \dfrac{4x}{x} + \dfrac{\sin 2x}{x}$

$\lim_{x \to 0} 4 + \lim_{x \to 0} \dfrac{\sin 2x}{x}$

$= 4 + 2 = \boxed{6}$

(15) $\lim_{x \to 0} \dfrac{\sin(2x + 2\Delta x) - \sin 2x}{\Delta x}$

Note: x, not Δx is the variable. Treat Δx as a constant

$= \dfrac{\sin(0 + 2\Delta x) - \sin 0}{\Delta x}$

$= \boxed{\dfrac{\sin(2\Delta x)}{\Delta x}}$

(16) $\lim_{x \to 4^+} \dfrac{3x - 12}{|8 - 2x|}$

* Pick any value $x > 4$ and plug in, say $x = 5$

$= \dfrac{3(5) - 12}{|8 - 2(5)|}$

$= \dfrac{3}{|-2|} = \boxed{\dfrac{3}{2}}$

(17) $\lim_{\theta \to 0} \dfrac{\sin^3 \theta}{\theta^2 (1 + \cos \theta)}$

$\lim_{\theta \to 0} \left(\dfrac{\sin \theta}{\theta}\right)\left(\dfrac{\sin \theta}{\theta}\right) \cdot \left(\dfrac{\sin \theta}{1 + \cos \theta}\right)$

$= 1 \cdot 1 \; \dfrac{0}{2}$

$= \boxed{0}$

(18) $\lim_{x \to -2} \frac{x^2-4}{x^3+2x^2+x+2}$

$\lim_{x \to -2} \frac{(\cancel{x+2})(x-2)}{(\cancel{x+2})(x^2+1)}$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$= \frac{-2-2}{(-2)^2+1} = \boxed{-\frac{4}{5}}$

(19) $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x}$

$\lim_{x \to \frac{\pi}{4}} \frac{1-\frac{\sin x}{\cos x}}{\sin x - \cos x} \left(\frac{\cos x}{\cos x}\right)$

$\lim_{x \to \frac{\pi}{4}} \frac{\cos x - \sin x}{(\sin x - \cos x)(\cos x)}$

$\lim_{x \to \frac{\pi}{4}} \frac{-\cancel{(\sin x - \cos x)}}{\cancel{(\sin x - \cos x)} \cdot \cos x}$

$\lim_{x \to \frac{\pi}{4}} \frac{-1}{\cos x}$

$\frac{-1}{\frac{1}{\sqrt{2}}} = \boxed{-\sqrt{2}}$

(20) $\lim_{x \to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$

$\lim_{x \to 1} \left(\frac{1}{x-1}\left(\frac{x+1}{x+1}\right) - \frac{2}{x^2-1}\right)$

$\lim_{x \to 1} \frac{x+1-2}{(x+1)(x-1)}$

$\lim_{x \to 1} \frac{\cancel{x-1}}{(x+1)(\cancel{x-1})} = \boxed{\frac{1}{2}}$

(21) $\lim_{x \to \frac{\pi}{3}} \frac{2\cos^2 x + 3\cos x - 2}{2\cos x - 1}$

$\lim_{x \to \frac{\pi}{3}} \frac{\cancel{(2\cos x - 1)}(\cos x + 2)}{\cancel{2\cos x - 1}}$

$\lim_{x \to \frac{\pi}{3}} \cos x + 2 = \frac{1}{2} + 2 = \boxed{\frac{5}{2}}$

(22) $\lim_{u \to \infty} \frac{4u^4+4}{(u^2-2)(2u^2-1)}$

$\lim_{u \to \infty} \frac{4u^4+4}{2u^4-5u^2+2} = \frac{4}{2} = \boxed{2}$

(23) $\lim_{x \to -4} \frac{(\cancel{x+4}) \cdot \ln(x+6)}{(\cancel{x+4})(x-4)}$

$\lim_{x \to -4} \frac{\ln(x+6)}{x-4} = \frac{\ln 2}{-8}$

$= -\frac{1}{8} \ln 2$

$= \frac{1}{8} \ln \frac{1}{2} = \ln \sqrt[8]{\frac{1}{2}}$

* log properties are fun!

(24) $\lim\limits_{x\to -2} \dfrac{\sin(x+2)}{x+2}$

* $\dfrac{\sin(x+2)}{x+2}$ is a shift

of $\dfrac{\sin x}{x}$ 2 units left

$\lim\limits_{x\to -2} \dfrac{\sin(x+2)}{x+2} = \boxed{1}$

(25) $\lim\limits_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4}$

even power function,

think of $f(x) = \frac{1}{x^2}$ at $x=0$.

$\lim\limits_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \boxed{\infty}$

(26) $\lim\limits_{x\to 2^+} \dfrac{x^3|\cancel{x-2}|}{\cancel{x-2}}$ both positive

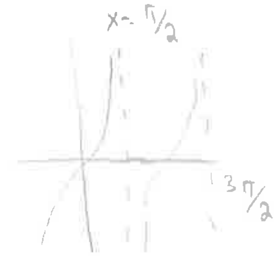
$\lim\limits_{x\to 2^+} x^3 = \boxed{8}$

(27) $\lim\limits_{x\to 3^+} \left(x-3-\boxed{\dfrac{1}{x-3}}\right)$

$= 3-3-\infty$

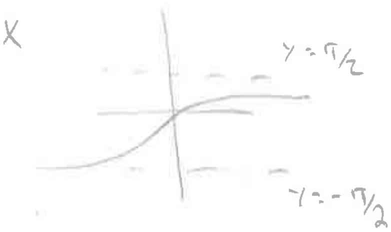
$= \boxed{-\infty}$

(28) $\lim\limits_{x\to \frac{\pi}{2}^+} \tan x$



$\boxed{-\infty}$

(29) $\lim\limits_{x\to \infty} \tan^{-1} x$



$= \boxed{\dfrac{\pi}{2}}$

(30) $1 \le f(x) \le x^2+2x+2$

$\lim\limits_{x\to -1} 1 = 1$

$\lim\limits_{x\to -1} x^2+2x+2 = 1$

by sandwhich (squeeze) them.

$\lim\limits_{x\to -1} f(x) = \boxed{1}$

(31) $3x \le f(x) \le x^3+2$

$\lim\limits_{x\to 1} 3x = 3$

$\lim\limits_{x\to 1} x^3+2 = 3$

$\therefore \lim\limits_{x\to 1} f(x) = \boxed{3}$

32) $\lim_{x \to a} \frac{2 \cdot f(x)}{h(x) - f(x)}$

$= \frac{2(-3)}{8-(-3)} = \boxed{-\frac{6}{11}}$

33) Look @ graph

a) 2-ish
b) -2
c) -5
d) -3
e) ∞
f) DNE
g) -3
h) 1

34) - 39) Look @ graph

34) 3
35) 2
36) ∞
37) DNE
38) 2
39) 0

40) $\lim_{x \to -1} \frac{x + x^2}{x^2 - 1}$

$\lim_{x \to -1} \frac{x(1+\cancel{x})}{(\cancel{x+1})(x-1)}$

$= \frac{-1}{-1-1} = \boxed{\frac{1}{2}}$

41) $\lim_{x \to \infty} \frac{(1-2x^2)^3}{(x^2+1)^3}$

$\lim_{x \to \infty} \left(\frac{-2x^2+1}{x^2+1}\right)^3$

$\left(\lim_{x \to \infty} \frac{-2x^2+1}{x^2+1}\right)^3$

$= (-2)^3 = \boxed{-8}$

42) $\lim_{x \to \infty} \left(\frac{1}{x} - \frac{x}{x-1}\right)$

$\lim_{x \to \infty} \frac{x - 1 - x^2}{x(x-1)}$

$\lim_{x \to \infty} \frac{-x^2+1-1}{x^2-x} = \boxed{-1}$

43) $\lim_{x \to 0} \frac{\frac{3}{x^2}}{\frac{2}{x^2} + \frac{105}{x}}$

$\lim_{x \to 0} \frac{3}{\cancel{x}\cancel{x}} \cdot \frac{\cancel{x^2}}{2+105x}$

$\lim_{x \to 0} \frac{3}{2+105x} = \boxed{\frac{3}{2}}$

(44) $\lim_{h\to 0} \frac{\sqrt{2+h+2} - \sqrt{4}}{h}$

$\lim_{h\to 0} \frac{\sqrt{4+h} - 2}{h}\left(\frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}\right)$

$\lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$

$= \frac{1}{\sqrt{4+0}+2} = \boxed{\frac{1}{4}}$

(45) $\lim_{n\to\infty} \frac{6n^2}{200-4n+kn^2} = \frac{6}{k} = \frac{1}{2}$

$k = 12$

(46) $\lim_{x\to\infty} \sqrt[3]{\frac{8+x^2}{x(x+1)}}$

$\sqrt[3]{\lim_{x\to\infty} \frac{x^2+8}{x^2+x}}$

$\sqrt[3]{1} = \boxed{1}$

(47) $\lim_{x\to -1} \frac{\sqrt{x^2+3} - 2}{x+1}$

$\lim_{x\to -1} \frac{\sqrt{x^2+3} - 2}{x+1}\left(\frac{\sqrt{x^2+3} + 2}{\sqrt{x^2+3} + 2}\right)$

$\lim_{x\to -1} \frac{x^2+3-4}{(x+1)(\sqrt{x^2+3}+2)}$

$\lim_{x\to -1} \frac{(\cancel{x+1})(x-1)}{(\cancel{x+1})(\sqrt{x^2+3}+2)}$

$= \frac{-2}{2+2} = \boxed{-\frac{1}{2}}$